# <u>Chapter 7</u>: Techniques of Integration

# <u>Section 7.3</u>: Trigonometric Substitution

<u>Idea</u>: We are going to be integrating functions that involve roots with addition or subtraction in them, like...

$$\int \sqrt{1 - x^2} \, dx \, \int \frac{x}{\sqrt{x^2 - 1}} \, dx \, \int \frac{x^3}{\sqrt{1 + x^2}} \, dx$$

Roots suck!!!

We want to get rid of the roots.

We'll use a trig. sub. (turns into a trig. integral) To decide which trig. sub. recall trig. identities Some Trig. Identities you need to know...

 $sin^2x + cos^2x = 1$  $\rightarrow sin^2 x = 1 - cos^2 x$  $\rightarrow cos^2 x = 1 - sin^2 x$  $tan^2x + 1 = sec^2x$  $\rightarrow tan^2 x = sec^2 x - 1$  $\rightarrow sec^2 x = tan^2 x + 1$ 

- <u>Idea</u>:  $\int \sqrt{1-x^2} \, dx$
- Trig. identities ...  $1 - cos^2 x = sin^2 x$  $1 - sin^2 x = cos^2 x$
- $sec^{2}x 1 = tan^{2}x$  $tan^{2}x + 1 = sec^{2}x$

# Use substitution ...

<u>Idea</u>:  $\int \frac{x}{\sqrt{x^2-1}} dx$ 

Trig. identities ...

 $1 - \cos^2 x = \sin^2 x$  $1 - \sin^2 x = \cos^2 x$  $sec^2x - 1 = tan^2x$  $tan^2x + 1 = sec^2 x$ 

Use substitution ...

<u>Idea</u>:  $\int \frac{x^3}{\sqrt{1+x^2}} dx$ 

Trig. identities ...  $1 - \cos^2 x = \sin^2 x$ 

- $1 \sin^2 x = \cos^2 x$  $sec^2x - 1 = tan^2x$
- $tan^2x + 1 = sec^2x$

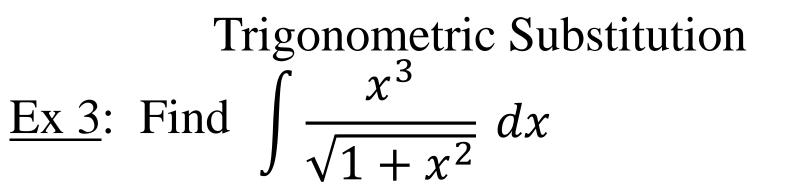
Use substitution ...

Notes:

- Make sure to find dx
- Make sure that your new integral only has  $\theta' s$  in it
- Your final answer should only have the original variable in it (you will probably have to use the triangle trick here)

Trigonometric Substitution  
Ex 1: Find 
$$\int \sqrt{1 - x^2} dx$$

# Trigonometric Substitution <u>Ex 2</u>: Find $\int \frac{x}{\sqrt{x^2 - 1}} dx$



<u>Issue #1</u>:

- Q: The number in front of the  $x^2$  term must be a 1. What if it isn't?
- <u>A</u>: Put an appropriate number in front of your trig sub to cancel out the number.

Ex 4: Find 
$$\int \sqrt{1 - 3x^2} \, dx$$

<u>Issue #2</u>:

- Q: The number being added inside the root must be a 1. What if it isn't?
- <u>A</u>: Put an appropriate number in front of your trig sub so you can factor out the number and end up with a 1 in its place.

Ex 5: Find 
$$\int \sqrt{5 - x^2} \, dx$$

## <u>Issue #1 and #2</u>:

- Q: What if the number in front of the  $x^2$  terms isn't a 1, and the number being added or subtracted isn't a 1?
- <u>A</u>: Put an appropriate number in front of your tirg sub so that...
- 1. The number in front of the  $x^2$  cancels out and changes to the number being added or subtracted.
- 2. Factor out the number being added or subtracted

<u>Issue #1 and #2</u>:

Ex 6: Find 
$$\int \sqrt{9 + 4x^2} \, dx$$

## <u>Issue #3</u>:

- <u>Q</u>: What if besides an number and an  $x^2$  terms, there is also an x terms inside the root?
- <u>A</u>: Complete the square

<u>Issue #3</u>:

- <u>A</u>: Complete the square
- 1. Separate the  $x \& x^2$  terms from the number term
- 2. Factor out the number in front of the  $x^2$  term
- 3. Complete the square on the parenthesis
- 4. Do I trig sub with an appropriate number in front of the trig function.

You may do something like... let  $x + 1 = \sqrt{2}tan\theta$ 

<u>Issue #1, #2, and #3</u>:

Ex 7: Find 
$$\int \frac{x^2}{(3+4x-4x^2)^{3/2}} dx$$